Robust Photon Entanglement via Quantum Interference in Optomechanical Interfaces

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Entanglement is often the key element in quantum information protocols. Here, we present schemes to generate robust photon entanglement in optomechanical interfaces via quantum interference. The schemes explore the excitation of the Bogoliubov dark mode and the destructive quantum interference between the bright modes of the interface, similar to electromagnetically induced transparency, to eliminate leading-order effects of the mechanical noise. Both continuous-variable and discrete-state entanglements that are robust against the mechanical noise can be achieved.

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Introduction. Optomechanical systems can serve as key elements in hybrid quantum networks that connect optical and microwave photons [1, 2]. The mechanical modes can couple with cavity photons of distinct frequencies, as was demonstrated in recent experiments of optomechanically induced transparencies, normal mode splitting, cavity cooling approaching the ground state, and *etc.* [3–16]. Schemes for quantum state manipulation and transfer in optomechanical interfaces have been intensively studied and the mechanical dark mode has recently been observed [17–27].

Entanglement is at the heart of many quantum information protocols. Various schemes for generating entanglement in optomechanical systems, either between the photon states or between the cavity and the mechanical modes, have been studied [28–44]. The amount of entanglement generated in these schemes is often limited by factors such as the stability of the system and the thermal noise of the mechanical mode. In stationary-state schemes where the cavities are continuously driven [28, 29, 31–34], the stability conditions place an upper bound for the effective optomechanical coupling and constrain the entanglement. In transient schemes where cavities are driven by pulses, the intrinsic nonlinearity of the radiation pressure force and the thermal noise can constrain the entanglement [41–44].

Continuous-variable entanglement between cavity modes can be generated by the parametric amplifier Hamiltonian $H_s = -g_s(a_1 a_2 + a_1^\dagger a_2^\dagger)$ with coupling g_s, where a_i ($i = 1, 2$) is the annihilation operator for mode i [45]. The cavity operators at time t can be written as $a_i(t) = \beta_i(r)$ in terms of the Bogoliubov modes

$$\beta_1(r) = \cosh(r)a_1 + i\sinh(r)a_2^\dagger \tag{1a}$$

$$\beta_2(r) = \cosh(r)a_2 + i\sinh(r)a_1^\dagger \tag{1b}$$

and the squeezing parameter $r = g_s t$. When applied to the vacuum state $|0_1 0_2\rangle$, this operation generates a two-mode squeezed vacuum state with the entanglement $E_N = 2r\log_2(e)$, where the entanglement is quantified by logarithmic negativity [46]. In optomechanical systems, cavity modes couple with mechanical modes but do not couple directly with each other. The optomechanical coupling can induce mixing between the cavity and the mechanical modes and expose the cavity modes to mechanical noise. In this work, we present schemes to generate strong photon entanglement that is robust against the mechanical thermal noise in an optomechanical interface by designing cavity operators in the time and frequency domains to have the Bogoliubov-like forms defined in Eqs.(1a,1b). In this interface, a Bogoliubov dark mode in the form of $\beta_2^\dagger(r)$ exists that does not contain mechanical operators. By combining the excitation of this mode and the destructive quantum interference between the bright modes of the interface, similar to electromagnetically induced transparency (EIT), the leading-order mechanical components can be eliminated from the cavity operators. Robust entanglement can hence be achieved between cavity photons in the time domain and between cavity outputs in the frequency domain. Meanwhile, we show that robust entanglement in discrete photon states $|\psi_{en}\rangle = (|0_1 1_2\rangle \pm |1_1 0_2\rangle)/\sqrt{2}$ can also be achieved in this interface. Our results demonstrate that optomechanical systems can act as noise-resilient hubs in hybrid quantum networks to perform key quantum operations such as quantum state transfer and entanglement generation. This facilitates the implementation of scalable hybrid quantum systems. Furthermore, the approaches used here can be extended to similar systems such as two cavity modes coupling with one noisy qubit to implement quantum operations.

Quantum interface with Bogoliubov dark mode. The interface is composed of two cavity modes coupling with the same mechanical mode with the interaction $\sum \hbar G_i a_i^\dagger a_i(b_m + b_m^\dagger)$, where b_m is the annihilation operator of the mechanical mode. One cavity is driven by red-detuned source with cavity detuning Δ_1 to generate anti-Stokes processes and the other cavity is driven by blue-detuned source with cavity detuning Δ_2 to generate Stokes processes as is illustrated in Fig.1(a,b). This model can be realized in many systems such as the hybrid system of a microwave cavity and an optical cavity both coupling with a membrane [31–34]. For generality, we use arbitrary units for the system parameters but

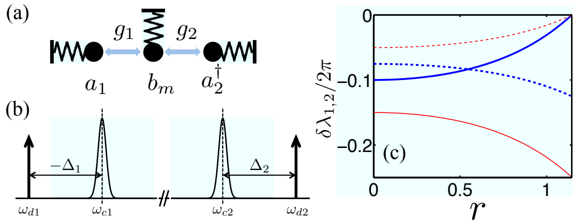

Figure 1: (a) The interface with three modes. (b) Cavity resonances at ω_{ci} and driving sources at ω_{di} with $\Delta_i = \omega_{di} - \omega_{ci}$. (c) $\delta\lambda_1$ (solid) and $\delta\lambda_2$ (dashed) versus r for $(\kappa_1, \kappa_2) = (0.3, 0.2)$ (thick) and $(0.2, 0.3)$ (thin) with $g_0 = 3$ and $\gamma_m = 0.001$ in arbitrary units.

choose realistic relative values between these parameters. Let the detunings be $-\Delta_1 = \Delta_2 = \omega_m$ with ω_m being the mechanical frequency. After standard linearization, the effective Hamiltonian in the interaction picture of $H_0 = \sum(-\hbar\Delta_i a_i^\dagger a_i) + \hbar\omega_m b_m^\dagger b_m$ can be written as

$$H_I = \hbar g_1(a_1^\dagger b_m + b_m^\dagger a_1) + i\hbar g_2(a_2^\dagger b_m^\dagger - a_2 b_m) \qquad (2)$$

where g_i's $(i = 1, 2)$ are the effective optomechanical couplings [47]. The environmental fluctuations can be represented by the input operators $a_{in}^{(i)}(t)$ for the cavities and $b_{in}(t)$ for the mechanical mode. The correlation functions for the input operators are $\langle a_{in}^{(i)}(t) a_{in}^{(i)\dagger}(t')\rangle = \delta(t-t')$ and $\langle b_{in}(t) b_{in}^\dagger(t')\rangle = (n_{th}+1)\delta(t-t')$ in the high temperature limit with thermal phonon number n_{th} [29]. The Langevin equation for this system is

$$i d\vec{v}(t)/dt = M\vec{v}(t) + i\sqrt{K}\vec{v}_{in}(t) \qquad (3)$$

with $\vec{v}(t) = [a_1(t), b_m(t), a_2^\dagger(t)]^{\mathrm{T}}$ for the system operators, $\vec{v}_{in}(t) = [a_{in}^{(i)}(t), b_{in}(t), a_{in}^{(2)\dagger}(t)]^{\mathrm{T}}$ for the input operators, diagonal matrix $K = \mathrm{Diag}[\kappa_1, \gamma_m, \kappa_2]$, and

$$M = \begin{pmatrix} -i\frac{\kappa_1}{2} & g_1 & 0 \\ g_1 & -i\frac{\gamma_m}{2} & ig_2 \\ 0 & ig_2 & -i\frac{\kappa_2}{2} \end{pmatrix}, \qquad (4)$$

where κ_i's (γ_m) are the cavity (mechanical) damping rates. With $\omega_m \gg g_i, \kappa_i, \gamma_i n_{th}$, the rotating wave approximation has been applied in the above equations.

The blue-detuned drive can induce instability in the interface, which affects the generation of entanglement. Using the Routh-Hurwitz criterion [48], we derive the stability conditions for this system which can be approximated as $g_1^2/g_2^2 > \max\{\kappa_2/\kappa_1, \kappa_1/\kappa_2\}$ in the strong coupling regime with $g_{1,2} \gg \kappa_i, \gamma_m$. This requires $g_1 > g_2$ for the system to be stable. Thus, we can write $g_1 = g_0\cosh(r)$ and $g_2 = g_0\sinh(r)$ with $r = \tanh^{-1}(g_2/g_1)$.

Let α_i be an eigenmode of the matrix M with eigenvalue λ_i. At zero damping, the eigenmodes are

$$\alpha_1 = \beta_2^\dagger;\ \alpha_{2,3} = (\beta_1 \pm b_m)/\sqrt{2} \qquad (5)$$

with eigenvalues $\lambda_1 = 0$ and $\lambda_{2,3} = \pm g_0$. The mode α_1 only contains the cavity modes and we call it the Bogoliubov dark mode. The modes $\alpha_{2,3}$ contain both cavity and mechanical modes and we call them the bright modes. An interesting feature is the symmetry of the bring modes, which gives $(\alpha_2 + \alpha_3)/\sqrt{2} = \beta_1$. At finite damping rates but in the strong coupling regime, the eigenmodes can be derived by treating the damping terms in the matrix M as perturbation. The eigenvectors of these modes form the matrix [23, 47]

$$U = \begin{pmatrix} -i\sinh(r) & \frac{\cosh(r)}{\sqrt{2}} + x_2 & \frac{\cosh(r)}{\sqrt{2}} - x_2 \\ x_1 & \frac{1}{\sqrt{2}} - x_3 & -\frac{1}{\sqrt{2}} - x_3 \\ \cosh(r) & \frac{i\sinh(r)}{\sqrt{2}} - x_4 & \frac{i\sinh(r)}{\sqrt{2}} + x_4 \end{pmatrix} \qquad (6)$$

where $x_i = O(\kappa_i/g_0, \gamma_m/g_0)$ is a first-order correction to the eigenmodes by the perturbation. The normalization conditions for the eigenvectors give $\sum_l U_{li}U_{lj} = \delta_{ij}$ to the first order of the perturbation. We also have

$$\alpha_1 = \beta_2^\dagger + x_1 b_m;\ (\alpha_2 + \alpha_3)/\sqrt{2} = \beta_1 - \sqrt{2}x_3 b_m \qquad (7)$$

which contain first-order terms $O(x_i)b_m$ of the mechanical mode. The eigenvalues are also modified by first-order corrections with $\lambda_1 = i\delta\lambda_1$ and $\lambda_{2,3} = \pm g_0 + i\delta\lambda_2$. The small imaginary parts $\delta\lambda_i$ are directly related to the stability conditions of the interface which becomes unstable when one of $\delta\lambda_i$ becomes positive.

Robust entanglement in time domain. Under the adiabatic condition for the couplings [47, 49], the Langevin equation can be written in terms of the eigenmodes as

$$i d\vec{\alpha}(t)/dt = \Lambda(t)\vec{\alpha}(t) + iU^{-1}(t)\sqrt{K}\vec{v}_{in}(t) \qquad (8)$$

where $\vec{\alpha} = [\alpha_1, \alpha_2, \alpha_3]^{\mathrm{T}}$ and $\Lambda(t) = \mathrm{Diag}[\lambda_1, \lambda_2, \lambda_3]$ with $\vec{\alpha}(t) = U^{\mathrm{T}}(t)\vec{v}(t)$ and $U^{\mathrm{T}}MU = \Lambda$. The operators $\vec{\alpha}(t)$ can be derived by integrating Eq.(8). At zero damping, $\alpha_1(t) = \alpha_1(0)$ and $\alpha_{2,3}(t) = \exp(\mp i\varphi(t))\alpha_{2,3}(0)$ with $\varphi(t) = \int_0^t dt' g_0(t')$. Applying Eq.(5), we derive that

$$\beta_1(t) = \beta_1(0)\cos\varphi(t) - ib_m(0)\sin\varphi(t) \qquad (9)$$

which mixes the cavity and the mechanical modes, and $\beta_2(t) = \beta_2(0)$. At time t_n with $\varphi(t_n) = n\pi$ for integer n, $\beta_1(t_n) = (-1)^n\beta_1(0)$. Hence, the Bogoliubov operators at time t_n only contain the cavity modes $a_i(0)$ with the mechanical mode eliminated by the destructive quantum interference between the phase factors in $\alpha_{2,3}(t)$ [50].

For time-independent couplings with g_0 and r, $\varphi(t) = g_0 t$. At $t_n = n\pi/g_0$ for even n, Eq.(9) gives $\beta_i(t_n) = \beta_i(0)$. Hence, $a_i(t_n) = a_i(0)$ and the cavities return to their initial state. For odd n, $\beta_1(t_n) = -\beta_1(0)$. The cavity operators at t_n can be derived as

$$\begin{pmatrix} a_1(t_n) \\ a_2^\dagger(t_n) \end{pmatrix} = \begin{pmatrix} \cosh(2r) & -i\sinh(2r) \\ i\sinh(2r) & \cosh(2r) \end{pmatrix} \begin{pmatrix} -a_1(0) \\ a_2^\dagger(0) \end{pmatrix} \qquad (10)$$

which gives a two-mode squeezed vacuum state with squeezing parameter $2r$ and entanglement $4r\log_2 e$ when applied to the vacuum state. These operators do not contain the mechanical mode $b_m(0)$ and the entanglement at t_n is thus not subject to the effect of thermal fluctuations in the initial mechanical state. We can also consider time-dependent couplings. For $g_1(t) = g_0\cosh(\lambda t)$ and $g_2(t) = g_0\sinh(\lambda t)$ under the adiabatic condition $\lambda \ll g_0$ [47], we derive that at $t_n = n\pi/g_0$ for integer n,

$$\begin{pmatrix} a_1(t_n) \\ a_2^\dagger(t_n) \end{pmatrix} = \begin{pmatrix} \cosh(r) & -i\sinh(r) \\ i\sinh(r) & \cosh(r) \end{pmatrix} \begin{pmatrix} (-1)^n a_1(0) \\ a_2^\dagger(0) \end{pmatrix} \tag{11}$$

which describes a two-mode squeezed vacuum state with squeezing parameter $r = \lambda t_n$ and does not contain the mechanical component $b_m(0)$.

At finite damping rates, both the thermal fluctuations in the initial mechanical state with average phonon number n_0 and the input noise of the mechanical bath with thermal excitation number n_{th} affect the entanglement generated in the time-domain schemes. By solving Eq.(8), the cavity operators $a_i(t_n)$ can be derived [47], which include terms of $O(\kappa_i/g_0)b_m(0)$ and terms of $O(\int dt'\sqrt{\gamma_m}b_{in}(t'))$ to leading order of κ_i/g_0 and γ_m/g_0. We first assume $n_0 = n_{th}$. At $t_n = n\pi/g_0$, the above terms affect the covariance matrix of the cavity state as $O(\kappa_i^2/g_0^2)n_{th}$ and $O(\gamma_m n_{th}/g_0)$ respectively. While at any other time $t \neq t_n$, the cavity operators contain the mechanical mode as $O(1)b_m(0)$ which affects the covariance matrix as n_{th}. The destructive quantum interference at t_n suppresses the thermal effects significantly by eliminating the leading order terms of $b_m(0)$ from the cavity operators. The entanglement at t_n is hence robust against the thermal noise. Note that other first-order terms in $a_i(t)$ include $O(\kappa_i/g_0)a_i(0)$ due to the decay of the eigenmodes and $O(\int dt'\sqrt{\kappa_i}a_{in}^{(i)}(t'))$ due to cavity input noise, both of which affect the covariance matrix as $O(\kappa_i/g_0)$.

The entanglement of the time-domain schemes is plotted in Fig.2(a,b) [47]. At finite temperature, sharp resonance peaks appear at t_n, where the peak values decrease slowly with n_{th} but the peak widths decrease quickly with n_{th}. This is because the cavity operators $a_i(t_n \pm \delta t)$ at the small deviate δt from t_n contain the mechanical mode as $O(g_0\delta t)b_m(0)$ as can be derived from Eq.(9). This term affects the covariance matrix and the entanglement as $(g_0\delta t)^2 n_{th}$ and narrows the resonance peaks for large n_{th}. In Fig.2(c), it is shown that the entanglement at the resonant peaks remains sizable even for $n_{th} \sim 10^4$, in sharp contrast to the stationary-state entanglement which decreases to zero quickly. For the above, n_0 is equal to n_{th}. In Fig.2(d), we study the effect of n_0 on the entanglement. For $n_0 = 0$, the entanglement increases with time. For finite n_0, the resonance peaks appear again and the peak widths decrease quickly with n_0. This result clearly verifies our analysis that the thermal fluctuations in initial mechanical state narrow the resonances.

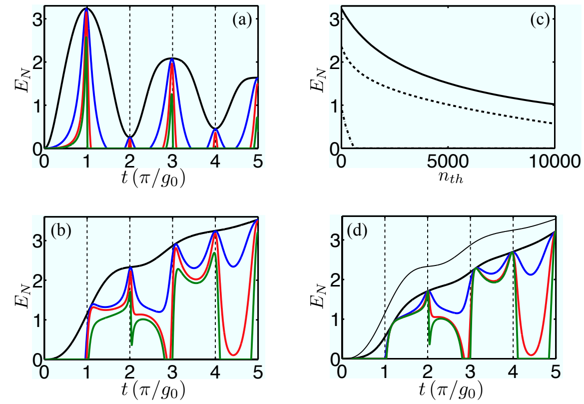

Figure 2: (Color online) Entanglement between cavity photons. (a) and (b) E_N versus time for time-independent scheme ($r = 1$) and adiabatic scheme with $r(t_2) = 1$; $n_0 = n_{th} = 0, 10, 10^2, 10^3$ from top to bottom. (c) E_N versus n_{th} for time-independent scheme at t_1 (solid), adiabatic scheme at t_2 (dashed), and stationary state (dash-dotted). (d) E_N for adiabatic scheme at t_2; $n_{th} = 10^3$ and $n_0 = 0, 10, 10^2, 10^3$ from top to bottom, and $n_0 = n_{th} = 0$ (thin). $(\kappa_1, \kappa_2) = (0.3, 02)$ and other parameters are same as in Fig.1.

Robust entanglement in frequency domain. For time-independent couplings, the Langevin equation can be written in the frequency domain as $\vec{v}(\omega) = (I\omega - M)^{-1}\sqrt{K}\vec{v}_{in}(\omega)$ with the frequency components defined by $\hat{O}(\omega) = \int dt\hat{O}(t)e^{i\omega t}/\sqrt{2\pi}$. Using the input-output relation, the output operators can be derived as $\vec{v}_{out}(\omega) = \widehat{S}(\omega)\vec{v}_{in}(\omega)$ with $\widehat{S}(\omega) = (I - i\sqrt{K}(I\omega - M)^{-1}\sqrt{K})$. To study the entanglement between cavity outputs, we define the operators $a_x^{(i)}(\omega_n) = \int d\omega g_\Delta(\omega - \omega_n)a_x^{(i)}(\omega)$ for $\omega_n = n\Delta\omega$ (n integer) and $x = in, out$ with the envelop function $g_\Delta(\omega) = 1/\sqrt{\Delta\omega}$ for $\omega \in (-\frac{\Delta\omega}{2}, \frac{\Delta\omega}{2})$ and $g_\Delta(\omega) = 0$ otherwise. The commutation relations for these operators are $[a_x^{(i)}(\omega_m), a_x^{(j)\dagger}(\omega_n)] = \delta_{mn}\delta_{ij}$, which ensures that the covariance matrix of their quadrature variables can be directly used to calculate entanglement [45, 47]. In Fig.3(a,b), entanglement between the output modes is plotted versus ω_n. Three resonance peaks appear at $\omega_n = 0, \pm g_0$, corresponding to the three eigenmodes α_i respectively. At ω_0 (*i.e.* ω_n for $n = 0$), the entanglement decreases slowly at large phonon number n_{th} and is robust against the thermal noise even for $n_{th} = 10^4$. While at $\omega_n = \pm g_0$, the entanglement decreases quickly to reach zero. This is illustrated in Fig.3(c,d).

For input noise at frequency ω_n, the excitations of eigenmodes are $\vec{\alpha}(\omega_n) = i(I\omega_n - \Lambda)^{-1}U^{\mathrm{T}}\sqrt{K}\vec{v}_{in}(\omega_n)$ [47]. At ω_0, the Bogoliubov dark mode is strongly excited as

$$\alpha_1(\omega_0) = \begin{pmatrix} \frac{\sinh(r)}{\delta\lambda_1} & i\frac{x_1}{\delta\lambda_1} & i\frac{\cosh(r)}{\delta\lambda_1} \end{pmatrix} \cdot \sqrt{K}\vec{v}_{in}(\omega_0) \tag{12}$$

with $x_1/\delta\lambda_1 \sim O(1/g_0)$. The bright modes are weakly excited as $\alpha_{2,3}(\omega_0) \propto 1/g_0$ with the relation

$$(\alpha_2(\omega_0) + \alpha_3(\omega_0))/\sqrt{2} = -\sqrt{\gamma_m}b_{in}(\omega_0)/g_0. \tag{13}$$

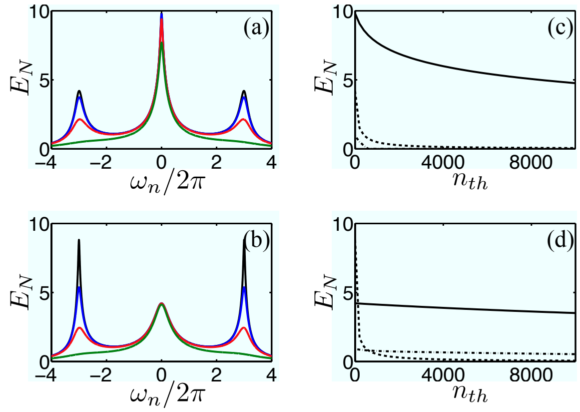

Figure 3: (Color online) Entanglement between output photons. (a) and (b) E_N versus ω_n for $(\kappa_1, \kappa_2) = (0.3, 0.2)$ and $(0.2, 0.3)$; $n_{th} = 0, 10, 10^2, 10^3$ from top to bottom. (c) and (d) E_N versus n_{th} for $(\kappa_1, \kappa_2) = (0.3, 0.2)$ and $(0.2, 0.3)$ at ω_0 (solid), $\omega_n = \pm g_0$ (dashed), and stationary state (dash-dotted). Other parameters are same as in Fig.1.

The cavity operators $a_i(\omega_0)$ can be derived from these results and contain the cavity inputs as $O(a_{in}^{(i)}(\omega_0)/\sqrt{\kappa_i})$ and the mechanical input as $O(\sqrt{\gamma_m} b_{in}(\omega_0)/g_0)$ [47]. The mechanical input is a factor κ_i/g_0 smaller than the cavity inputs when $\gamma_m n_{th} \sim \kappa_i$. This is due to the destructive quantum interference between the excitations of $\alpha_2(\omega_0)$ and $\alpha_3(\omega_0)$. As a result, the thermal effect in the covariance matrix of the output operators $a_{out}^{(i)}(\omega_0)$ is suppressed by a factor of $(\kappa_i/g_0)^2$ and the entanglement at ω_0 is robust against the mechanical thermal noise. At $\omega_n = g_0$, $\alpha_{1,3}(g_0) \sim 1/g_0$ are weakly excited. While one of the bright modes $\alpha_2(g_0) \sim 1/\delta\lambda_2$ is strongly excited due to its resonance with the input noise and contains the mechanical input as $O(\sqrt{\gamma_m} b_{in}(g_0)/\delta\lambda_2)$. This large mechanical term cannot be eliminated from $a_{out}^{(i)}(g_0)$ and will impair the entanglement quickly as n_{th} increases. Similar results can be derived for $\omega_n = -g_0$.

The entanglement here depends strongly on the cavity damping rates $\kappa_{1,2}$ as is shown in Fig.3(a,b). This is due to the dependence of $\delta\lambda_i$ on the damping rates. In the stable regime, $\delta\lambda_{1,2} < 0$. For $\kappa_1 > \kappa_2$, as the squeezing parameter r increases towards the unstable regime, $|\delta\lambda_1|$ becomes much smaller than $|\delta\lambda_2|$ with $\delta\lambda_1 \to 0$ as is plotted in Fig.1(c). In this regime, the excitation of the Bogoliubov dark mode at ω_0 becomes significantly stronger than the excitation of the bright modes at $\omega_n = \pm g_0$ so that the entanglement at ω_0 is stronger. For $\kappa_2 > \kappa_1$, as r increases towards the unstable regime, $\delta\lambda_2 \to 0$ and the entanglement at $\omega_n = \pm g_0$ is stronger. Meanwhile, regardless of the damping rates, the entanglement at ω_0 is always robust against the thermal noise. This tells us that in a hybrid interface with very different cavity damping rates, by applying red-detuned drive to the cavity mode with larger damping rate and applying blue-detuned drive to the other cavity to get $\kappa_1 > \kappa_2$, stronger entanglement that is robust against thermal noise can be achieved.

Robust entanglement in discrete states. This interface can also be used to generate entanglement in discrete states such as $(|1_1 0_2\rangle \pm |0_1 1_2\rangle)/\sqrt{2}$ [51]. Let the two cavities both be driven by red-detuned sources with $-\Delta_i = \omega_m$. In [22, 23], this setup was studied for high fidelity transfer of quantum states. Assume the couplings to be $g_1(t) = g_0 \sin(\lambda t)$ and $g_2(t) = -g_0 \cos(\lambda t)$, varying adiabatically with $\lambda < g_0$. At $t_f = \pi/4\lambda$ and by choosing $\lambda = g_0/4n$ for integer n, the cavity operators become

$$\begin{pmatrix} a_1(t_f) \\ a_2(t_f) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a_1(0) \\ (-1)^n a_2(0) \end{pmatrix}. \quad (14)$$

It can be proven that for an initial cavity state $|1_1 0_2\rangle$, the final state of the cavities is $|\psi_{en}\rangle = (|1_1 0_2\rangle + |0_1 1_2\rangle)/\sqrt{2}$. Similarly, for the initial state $|0_1 1_2\rangle$, the final state is $|\psi_{en}\rangle = (|1_1 0_2\rangle - |0_1 1_2\rangle)/\sqrt{2}$. The effect of the thermal noise can be studied by solving the Langevin equation using perturbation theory [47]. The cavity operators $a_i(t_f)$ contain the mechanical mode as $O(\kappa_i/g_0) b_m(0)$ which is suppressed by factor κ_i/g_0 due to the destructive interference. Hence, the discrete-state entanglement is also robust against thermal noise.

Conclusions. We study an optomechanical interface for the generation of photon entanglement that is robust against the mechanical noise in both the time and the frequency domains. Due to the excitation of the Bogoliubov dark mode and the quantum interference between the bright modes, the effect of the mechanical noise is significantly suppressed. Both continuous-variable and discrete-state entanglements can be generated. When combined with the state transfer schemes, this quantum interface provides a promising building block for hybrid quantum networks and for quantum state engineering.

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